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Kinetic Formulation of the Isentropic Gas Dynamics and p-Systems

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Abstract: We consider the 2×2 hyperbolic system of isentropic gas dynamics, in both Eulerian or Lagrangian variables (also called the p-system). We show that they can be reformulated as a kinetic equation, using an additional kinetic variable. Such a formulation was first obtained by the authors in the case of multidimensional scalar conservation laws. A new phenomenon occurs here, namely that the advection velocity is now a combination of the macroscopic and kinetic velocities. Various applications are given: we recover the invariant regions, deduce new L^∞ estimates using moments lemma and prove $L^\infty - w*$ stability for $\gamma \geq 3$.

Introduction

We consider the equations of isentropic gas dynamics. In the Eulerian coordinates these equations form a 2×2 hyperbolic system of nonlinear conservation laws

$$\begin{cases} \partial_t \varrho + \partial_x \varrho u = 0\,, \\ \partial_t(\varrho u) + \partial_x(\varrho u^2 + p(\varrho)) = 0,\ t \geq 0,\ x \in \mathbb{R}\,, \\ p(\varrho) = \kappa p^\gamma,\ \gamma > 1,\ \kappa = \dfrac{(\gamma-1)^2}{4\gamma}\,, \end{cases} \tag{1}$$

where the unknowns $\varrho(t,x)$ and $q := \varrho u(t,x)$ are respectively the density and the momentum of the gas. They are given at time $t = 0$ by the initial data $\varrho^0(x)$ and $q^0 = \varrho^0 u^0(x)$. And of course, $\varrho \geq 0$ on $\mathbb{R}^+ \times \mathbb{R}$.

We will also consider another 2×2 system,

$$\begin{cases} \partial_t v - \partial_x w = 0\,, \\ \partial_t w + \partial_x p(v) = 0,\ t \geq 0,\ x \in \mathbb{R}\,, \end{cases} \tag{2a}$$

endowed with the pressure law

$$p(v) = \kappa v^{-\gamma}\,, \qquad \gamma > 0\,, \qquad \kappa = \frac{(\gamma-1)^2}{4\gamma}\,. \tag{2b}$$