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P-Determinant Regularization Method for Elliptic Boundary Problems

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Abstract. An expression for the p-determinant of the quotient of two differential elliptic operators with boundary conditions is given in terms of the boundary values of their solutions. Applications to physical examples are considered.

1. Introduction

An expression for the Fredholm determinant of the quotient of two elliptic operators defined on a closed manifold with boundary in terms of pseudodifferential operators defined on the boundary was given by Forman in [5]. In this paper, we aim to establish an analogous expression for the so called p-determinant of the quotient of the operators holding even in the case where it has not Fredholm determinant. This case is usually found in Quantum Physics where the p-determinant can be taken as a regularization technique for divergent determinants [9]. In order to describe it, let us recall some definitions.

A compact operator A defined on a Hilbert space H is an element of the p^{th} Schatten class $\mathscr{I}_p$, for $p \geq 1$ an integer, if $|A|^p$ is a trace class operator, i.e. if

$$\operatorname{Tr}(|A|^p) = \sum_{j=1}^{\infty} \mu_j^p(A) < \infty\,,$$

where $\mu_j(A)$, the singular values of A, are the eigenvalues of $|A| = \sqrt{A^*A}$. In particular $\mathscr{I}_1$ and $\mathscr{I}_2$ are the ideals of trace class and Hilbert-Schmidt operators on H. If I denotes the identity operator on H, the Fredholm determinant, $\det_1(I - A)$, is defined as $\prod_{j=1}^{\infty}(1 - \lambda_j)$, where $\{\lambda_j\}_j$ denotes the proper values of A when A is a trace class operator. The p-determinant of $I - A$ is defined, for $A \in \mathscr{I}_p$, as [6, 4, 9]:

$$\det_p(I - A) = \det_1\left\{I - (I - A)\exp\left[A + \frac{A^2}{2} + \ldots + \frac{A^{p-1}}{p-1}\right]\right\},$$