

On the Classification of Simple Vertex Operator Algebras

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Abstract: Inspired by a recent work of Frenkel–Zhu, we study a class of (pre-)vertex operator algebras (voa) associated to the self-dual Lie algebras. Based on a few elementary structural results we propose that \mathcal{V} , the category of \mathbf{Z}_+ -graded prevoas V in which $V[0]$ is one-dimensional, is a proper setting in which to study and classify simple objects. The category \mathcal{V} is organized into what we call the minimal k^{th} types. We introduce a functor Γ – which we call the Frenkel–Lepowsky–Meurman functor – that attaches to each object in \mathcal{V} a Lie algebra. This is a key idea which leads us to a (relative) classification of the *simple minimal first type*. We then study the set of all Virasoro structures on a fixed minimal first type V , and show that they are in turn classified by the orbits of the automorphism group $\text{Aut}(\Gamma(V))$ in $\text{cent}(\Gamma(V))$. Many new examples of voas are given. Finally, we introduce a generalized Kac–Casimir operator and give a simple proof of the irreducibility of the prolongation modules over the affine Lie algebras.

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