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A Symmetric Family of Yang–Mills Fields

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Abstract: We examine a family of finite energy $SO(3)$ Yang–Mills connections over S^4, indexed by two real parameters. This family includes both smooth connections (when both parameters are odd integers), and connections with a holonomy singularity around 1 or 2 copies of RP^2. These singular YM connections interpolate between the smooth solutions. Depending on the parameters, the curvature may be self-dual, anti-self-dual, or neither. For the (anti)self-dual connections, we compute the formal dimension of the moduli space. For the non-self-dual connections we examine the second variation of the Yang–Mills functional, and count the negative and zero eigenvalues. Each component of the non-self-dual moduli space appears to consist only of conformal copies of a single solution.

1. Introduction and Statement of Results

1.1 Main Results. Until recently, the phrase "Yang–Mills theory in four dimensions" essentially meant the study of smooth solutions to the (anti) self-duality equations

$$*F = \pm F\,, \tag{1.1}$$

where F is the curvature of a connection A, usually with gauge group $SU(2)$ or $SO(3)$, on a bundle over a Riemannian 4-manifold M, which may or may not have a boundary. The moduli space of such solutions, up to gauge invariance, gives topological information about M, a fact which was exploited by Donaldson and others to make tremendous progress in the topology of 4-manifolds (see [DK] for an overview).

In recent years the field has expanded in two directions. First, there is the study of nonself-dual Yang–Mills connections. These are solutions to the full Yang–Mills equations,

$$d_A^* F = 0\,, \tag{1.2}$$

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