Commun. Math. Phys. 163, 245–255 (1994)

Communications in
Mathematical
Physics

Asymptotics Beyond All Orders and Analytic Properties of Inverse Laplace Transforms of Solutions

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Received: 1 February 1993/in revised form: 20 December 1993

Abstract: A number of modern mathematical and physical problems require the study of delicate asymptotic properties lying "beyond" the power series asymptotics. In this paper we suggest a link between these asymptotic problems and some analytic properties of inverse Laplace transforms of the corresponding solutions. The main result claims that these inverse transforms are holomorphic in an appropriately cut complex plane. A direct consequence of this is the nonexistence of solutions to the class of "asymptotics beyond all orders" problems, such as regular shocks of the Kuramoto–Sivashinsky equation ([Gr]), needle crystal solutions of the simple geometrical model of crystal growth ([KS]), solitary wave solutions to a class of the fifth-order Kortveg–de Vries equations ([KO, Sect.8], [GJ]), homoclinic orbits of some singularly perturbed mappings ([Ec, HM]) and others.

I. Introduction

Let us consider a differential equation of the type

$$x^{1-r}y'(x) = f(x,y), \quad x \in \overline{C},\ y \in C^n,\ r \in N\,, \tag{1.1}$$

where the vector-valued function $f(x,y)$ is assumed to be homomorphic at $(\infty, 0) \in \overline{C} \times C^n$. The particular equations

$$v'''(x) + v'(x) + v^2(x) = 0, \tag{1.2}$$

$$v'''(x) + v'(x) + \frac{e^{v(x)}}{x} = \frac{1}{x} + \frac{2}{x^3}, \tag{1.3}$$

★ Supported by a grant of the Ministry of Science and Technology of Israel.
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