Commun. Math. Phys. 163, 217–244 (1994)

Communications in Mathematical Physics

Selberg Trace Formula for Bordered Riemann Surfaces: Hyperbolic, Elliptic and Parabolic Conjugacy Classes, and Determinants of Maass–Laplacians

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Received: 24 August 1992/in revised form: 22 October 1993

Abstract: The Selberg trace formula for automorphic forms of weight $m \in \mathbb{Z}$, on bordered Riemann surfaces is developed. The trace formula is formulated for arbitrary Fuchsian groups of the first kind with reflection symmetry which include hyperbolic, elliptic and parabolic conjugacy classes. In the case of compact bordered Riemann surfaces we can explicitly evaluate determinants of Maass–Laplacians for both Dirichlet and Neumann boundary-conditions, respectively. Some implications for the open bosonic string theory are mentioned.

I. Introduction

Spectral theory of automorphic forms has recently seen some activity in the physical literature, due to its importance in string theory. This theory originates from the work of A. Selberg [50], where the famous so-called "Selberg trace formula" was first presented. Other classical contributions are due to Hejhal [32, 33] and Venkov [56, 57]. String theory gave new interest in this work, first to refine the Selberg trace formula further in order to calculate determinants of Laplacians on Riemann surfaces, and second to develop other versions of the Selberg trace formula. Here the generalization to the fermionic- (super-) string theory was most important, leading to a formulation of a trace formula on super Riemann surfaces, the Selberg super trace formula [8, 25–27].

Because the original contribution of Selberg was founded in the field of number theory, physicists only lately acknowledged its value in periodic orbit theory as founded by Gutzwiller [30, 52] (see also Albeverio et al. for a thorough mathematical treatment of a particular system [1]), who rediscovered the Selberg trace formula years later within his formalism [31], and in quantum field theory on Riemann surfaces, i.e. the Polyakov approach [17–19, 47] to (bosonic-, fermionic- and super-) string theory. In the perturbative expansion of the Polyakov path

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