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$N = 2$ Topological Yang-Mills Theory on Compact Kähler Surfaces

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Abstract: We study a topological Yang-Mills theory with $N = 2$ fermionic symmetry. Our formalism is a field theoretical interpretation of the Donaldson polynomial invariants on compact Kähler surfaces. We also study an analogous theory on compact oriented Riemann surfaces and briefly discuss a possible application of Witten's non-Abelian localization formula to the problems in the case of compact Kähler surfaces.

1. Introduction

Several years ago, Witten introduced the topological Yang-Mills theory (TYMT) [1] on general 4-manifolds to provide a quantum field theoretical interpretation of the Donaldson polynomial invariants [2]. The basic property of the TYMT is that there is a fermionic symmetry which localizes the path integral to an integral over the moduli space $\mathscr{M}$ of anti-self-dual (ASD) connections. Geometrically, the fermionic operator δ_W acts on $\mathscr{M}$ as the exterior derivative. The action functional of the TYMT can be written as an δ_W-exact form,

$$S_W = \delta_W V \,. \tag{1.1}$$

In the TYMT, correlation functions of physical observables correspond to the Donaldson polynomial invariants.

The moduli space $\mathscr{M}$ of ASD connections on a compact Kähler surface has natural complex and Kähler structures [3], which implies that the TYMT has actually $N = 2$ fermionic symmetry generated by the holomorphic and the anti-holomorphic parts of δ_W, i.e. $\delta_W = \mathbf{s} + \bar{\mathbf{s}}$. Geometrically, we can interpret $\bar{\mathbf{s}}$ as the Dolbeault cohomology operator on $\mathscr{M}$. Then, the $N = 2$ version of the topological action may be written as

$$S = \mathbf{s}\bar{\mathbf{s}}\mathbf{B}_\mathbf{T} \,. \tag{1.2}$$

In the first part of this paper, we study TYMT on compact Kähler surfaces with $N = 2$ fermionic symmetry. In Sect. 2, we briefly sketch Donaldson theory and the TYMT of Witten in order to make this paper reasonably self-contained and to set