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The Newtonian Limit for Asymptotically Flat Solutions of the Vlasov-Einstein System

Alan D. Rendall

Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Strasse 1,
D-85740 Garching bei München, Germany
Fax: 89 32993235; E-mail: anr@cvxastro.mpa-garching.mpg.de

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Abstract: It is shown that there exist families of asymptotically flat solutions of the Einstein equations coupled to the Vlasov equation describing a collisionless gas which have a Newtonian limit. These are sufficiently general to confirm that for this matter model as many families of this type exist as would be expected on the basis of physical intuition. A central role in the proof is played by energy estimates in unweighted Sobolev spaces for a wave equation satisfied by the second fundamental form of a maximal foliation.

1. Introduction

It is a well known empirical fact that in many situations a general relativistic description of the motion of self-gravitating matter can be replaced to a good approximation by a non-relativistic, Newtonian one. In the usual formulation of Newtonian gravity the interaction is described by a single scalar function, the Newtonian potential. The relation of this to general relativity, where the fundamental object is a Lorentz metric, is obscure. The basic idea required to understand this relation mathematically was provided by Cartan [4]. He showed that Newtonian theory can be formulated in such a way that the basic object is an affine connection whose non-zero components are components of the gradient of the Newtonian potential. The role of the potential itself is then merely that of providing a convenient representation of this connection in certain coordinate systems. It was realised by Friedrichs [12] that the natural way to connect the two theories is to require that the Levi-Civita connection of the spacetime metric go over in the limit as the speed of light c goes to infinity into the connection defined by Cartan. Since then many authors have extended this work on the relations between the equations of the two theories and the physical interpretations of their solutions. This knowledge has been systematised in the frame

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