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The Jones-Witten Invariant for Flows on a 3-Dimensional Manifold

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Abstract: The Jones-Witten functional for knots is generalized for smooth flows on 3-dimensional manifolds. Explicit computations for the abelian case are given.

0. Introduction

Recently Witten has proposed a generalization of the Jones polynomial for links which has the advantage of being intrinsically defined on any closed orientable 3-manifold (see [W] and [At]). Witten defines his invariant via holomorphic sections of a vector bundle on a canonical moduli space related to a Heegard splitting of the 3-manifold. He develops this theory from the Hamiltonian point of view, and then he gives an interpretation of the Jones polynomial in terms of a 3-dimensional Yang-Mills theory, where the Lagrangian $\mathscr{L}$ is the functional which assigns to each connection its Chern-Simons character, with weight k (an integer or half integer for the $SU(N)$ or the $U(1)$ theories respectively):

$$\mathscr{L}:\mathscr{A} \to \mathbb{R}/\mathbb{Z},$$

$$\mathscr{L}_k(A) = \frac{k}{8\pi}\int_M tr\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right),$$

where $\mathscr{A}$ is the space of all connections on the trivial principal bundle $P = M\times G \to M$, and the structure group is either $SU(N)$ or $U(1)$. The trace corresponds to the bilinear Killing form for the compact gauge group under some explicit representation of the group.

Our purpose in this note is to extend Witten's ideas to construct new topological invariants of a smooth dynamical system, namely invariants under differentiable equivalence. A possible physical interpretation of our formula case is as an averaged Bohm-Aharonov effect for a "continuous" flux of electrons.