Commun. Math. Phys. 163, 17–32 (1994)

Communications in
Mathematical
Physics
© Springer-Verlag 1994

BRST Cohomology for Certain Reducible Topological Symmetries

Sophie Chemla, Jaap Kalkman

Rijkuniversiteit Utrecht, Mathematisch Instituut, Postbus 80.010, 3508 TA Utrecht, The Netherlands. e. mail: chemla@math.ruu.nl and kalkman@math.run.nl

Received: 15 March 1993/in revised form: 19 July 1993

Abstract: In this paper, two different definitions of the BRST complex are connected. We obtain the BRST complex of topological quantum field theories (leading to equivariant cohomology) from the standard definition of the classical BRST complex (leading to Lie algebra cohomology) provided that we include ghosts for ghosts. Hereby, we use a finite dimensional model with a semi-direct product action of $H \rtimes \mathrm{Diff}M$ on a configuration space M, where H is a compact Lie group representing the gauge symmetry in this model.

1. Introduction

It is well known that the BRST cohomology of a field theory with gauge group G equals the Lie algebra cohomology of Lie (G) with values in the functionals on the space of fields. Several years ago it was shown by Henneaux and others ([H, K-S]) that this is also true for the BRST cohomology of finite dimensional Hamiltonian systems. In the case where the G-action is not free on some open set, the differential algebra of the BRST complex has to be enlarged with so-called ghosts for ghosts to obtain cohomology classes with a physical meaning (see e.g, [F-H-S-T]).

On the other hand it is well known that the BRST cohomology of cohomological field theories ([Wi]) with additional gauge symmetries equals equivariant cohomology, thus giving the cohomology of all kinds of moduli spaces. However, as described in [O-S-vB] and [Ka], the differential leading to this cohomology is not the standard equivariant differential.

This paper grew out from the uneasy feeling that it is not good to have several disconnected definitions of the BRST complex. In the subsequent sections we will link the two different definitions mentioned above. Furthermore, we will explain the non-standard equivariant differential using results of [F-H-S-T]. To obtain this, we start from a G-action on a manifold M, where G is the semidirect product of a compact Lie group H and $\mathrm{Diff}(M)$ (the group of diffeomorphisms of M). This theory is topological in the sense that there is only one gauge orbit.