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An Explicit Description of the Global Attractor of the Damped and Driven Sine-Gordon Equation

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Abstract: We prove that the size of the finite-dimensional attractor of the damped and driven sine-Gordon equation stays small as the damping and driving amplitude become small. A decomposition of finite-dimensional attractors in Banach space is found, into a part $\mathscr{B}$ that attracts all of phase space, except sets whose finite-dimensional projections have Lebesgue measure zero, and a part $\mathscr{C}$ that only attracts sets whose finite-dimensional projections have Lebesgue measure zero. We describe the components of the $\mathscr{B}$-attractor and $\mathscr{C}$, which is called the "hyperbolic" structure, for the damped and driven sine-Gordon equation. $\mathscr{B}$ is low-dimensional but the dimension of $\mathscr{C}$, which is associated with transients, is much larger. We verify numerically that this is a complete description of the attractor for small enough damping and driving parameters and describe the bifurcations of the $\mathscr{B}$-attractor in this small parameter region.

Contents

1. Introduction

Dissipative nonlinear wave equations on a finite domain possess finite-dimensional attractors and if the equations are driven by an autonomous force a complete

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