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Self Duality of the Gauge Field Equations and the Cosmological Constant

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Abstract: This paper considers the Einstein equations coupled with the nonabelian gauge and Higgs fields. It is shown that, when cosmic string solutions are sought in the Einstein–Georgi–Glashow system and the Einstein–Weinberg–Salam system governing the gravitational-electromagnetic-weak interaction forces, the self duality conditions lead to positive values of the cosmological constant which can be expressed by some fundamental parameters in particle physics.

1. Introduction

In quantum field theory, phase transitions are described by a generalized order parameter, called the Higgs field, which is defined on spacetime and takes values in a range space. The spacetime symmetry, or the external symmetry, gives rise to Einstein's theory of general relativity or the theory of gravitation while the range space symmetry, or the internal symmetry, leads to the Yang–Mills–Higgs gauge theory or the field theory of electromagnetic and nuclear (strong and weak) forces. Therefore the coupling of the Einstein and the Yang–Mills–Higgs theories should naturally lead to a unified theoretical framework to house gravitational, electromagnetic, and nuclear forces. In fact, recent developments in cosmology and particle physics have already witnessed an exciting interaction of these two traditionally different areas and a lot of progress has been made in understanding some important issues. For example, it has been recognized that, due to the spontaneously broken symmetry, the coupled Einstein–Yang–Mills–Higgs equations may provide a class of interesting solutions called topological defects. The symmetry-breaking scales are realized by the gauge groups corresponding to various stages of the phase transitions after the Big Bang. These stable defects may be domain walls, monopoles, or strings but the former two types of solutions are disastrous for cosmological models and only the string solutions can lead to

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