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The Existence of Non-Minimal Solutions of the Yang-Mills-Higgs Equations Over R^3 with Arbitrary Positive Coupling Constant

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Abstract: This paper proves the existence of a non-trivial critical point of the $SU(2)$ Yang-Mills-Higgs functional on R^3 with arbitrary positive coupling constant. The critical point lies in the zero monopole class but has action bounded strictly away from zero.

1. Introduction

This paper establishes the existence of a non-globally-minimizing critical point with monopole number zero for the $SU(2)$ Yang-Mills-Higgs equations on R^3 with positive coupling constant λ. The Yang-Mills-Higgs equation on R^3 are a system of second order non-linear equations:

$$* D_A * F = [D_A\phi, \phi] \qquad \text{YMH}\lambda(1)\,,$$

$$* D_A * D_A\phi = \frac{\lambda}{2}\phi(|\phi|^2 - 1) \qquad \text{YMH}\lambda(2)\,.$$

Here, the variables are A, a connection on a principal $SU(2)$ bundle and ϕ, a section of the vector bundle $E = su(2) \times R^3$ called the Higgs field. D_A is covariant differentiation and F is the curvature of the connection A, $F = dA + A \wedge A$.

These equations can be viewed as the variational equations of the action functional:

$$\mathbf{A}(A, \phi) = \frac{1}{2}\|F_A\|_2^2 + \frac{1}{2}\|D_A\phi\|_2^2 + \frac{\lambda}{8}\|\, |\phi|^2 - 1\|_2^2\,.$$

If we restrict to finite action solutions, then the Higgs field approaches an asymptotic limit. Namely we have

$$\lim_{|x|\to\infty} |\phi(x)| = 1\,,$$

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