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A Nonlinear Instability for 3 × 3 Systems of Conservation Laws

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Abstract: The phenomenon of nonlinear resonance provides a mechanism for the unbounded amplification of small solutions of systems of conservation laws. We construct spatially 2π-periodic solutions $u^N \in C^\infty([0,\ t_N] \times \mathbb{R})$ with t_N bounded, satisfying

$$\|u^N\|_{L^\infty([0,\ t_N]\times\mathbb{R})} \to 0, \quad \int_0^{2\pi} |\partial_x u^N(0,x)|\, dx \leqq C\,,$$

$$\int_0^{2\pi} |\partial_x u^N(t_N,x)|\, dx \geqq N, \quad \|u^N(t_N,x)\|_{L^p(\mathbb{R})} \geqq N \|u^N(0,x)\|_{L^p(\mathbb{R})} \quad 1 \leqq p \leqq \infty\,.$$

The variation grows arbitrarily large, and the sup norm is amplified by arbitrarily large factors.

Outline.

1. Main Result

The main existence theorems for $k \times k$ systems of conservation laws [G, CS, GL, NS, D, Y], have a common feature: either the systems under consideration have $k \leqq 2$, or the initial data are of small total variation. In the latter cases, the variation is uniformly bounded by a fixed multiple of the initial variation. In this note we explain that these restrictions are essential. When $k \geqq 3$, nonlinear resonance is a