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# Large Deviations, Averaging and Periodic Orbits of Dynamical Systems

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**Abstract:** The paper deals with large deviation bounds for the proportion of periodic orbits with irregular behavior for expansive dynamical systems with specification, in particular, we obtain estimates for large deviations from the equidistribution for closed geodesics on negatively curved manifolds. We derive also large deviation bounds in the averaging principle when the fast motion is the shift along periodic orbits.

## 1. Introduction

Let $f^t: M \to M$ be a group of homeomorphisms of a compact metric space with either discrete time $t \in \mathbb{Z}$ or continuous time $t \in \mathbb{R}$. A point $x \in M$ is called periodic if $f^t x = x$ for some $t > 0$ and the orbit $\{f^s x,\, s \in \mathbb{Z}$ or $s \in \mathbb{R}\}$ of such $x$ is called a closed (or periodic) orbit containing $x$. Denote by $CO$ the set of all closed orbits and by $CO_\delta(t)$ those orbits from $CO$ with some period in the interval $[t-\delta,\, t+\delta]$. Let $\gamma \in CO$, $x \in \gamma$, and $\tau(\gamma)$ denotes the least period of $\gamma$. Then the map $t \to f^t x$ sends the Lebesgue measure on $[0, \tau(\gamma)]$ to the measure $\tau(\gamma)\zeta_\gamma$ on $\gamma$ where $\zeta_\gamma = (\tau(\gamma))^{-1} \int_0^{\tau(\gamma)} \delta_{f^s x}\, ds$ in the continuous time case and $\zeta_\gamma = (\tau(\gamma))^{-1} \sum_{i=1}^{\tau(\gamma)} \delta_{f^i x}$ in the discrete time case, with $\delta_y$ standing for the unit mass at $y$. Set $\mu_{t,\delta} = N_{t,\delta}^{-1} \sum_{\gamma \in CO_\delta(t)} \zeta_\gamma$, where $N_{t,\delta} = \# \{CO_\delta(t)\}$ is the number of elements in $CO_\delta(t)$ which is finite if $f^t$ is an expansive dynamical system (see, for instance, [BW]). By [B1, B2, and B3] (see also [F] and [Pa]) $\mu_{t,\delta}$ weakly converges as $t \to \infty$ to the measure $\mu_{\max}$ with maximal entropy for $f^t$ provided $f^t$ is a hyperbolic dynamical system and, in fact, the more general conditions of expansiveness and specification will suffice. This was called by R. Bowen the equidistribution of closed orbits. For $\Gamma \subset CO$ set $\nu_{t,\delta}(\Gamma) = N_{t,\delta}^{-1} \# \{\Gamma \cap CO_\delta(t)\}$. Then by [La],

$$\lim_{t \to \infty} \nu_{t,\delta}\{\gamma \in CO:\, \zeta_\gamma \notin U_{\mu_{\max}}\} = 0 \tag{1.1}$$

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