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The Inverse Scattering Problem at Fixed Energy for the Three-Dimensional Schrödinger Equation with an Exponentially Decreasing Potential

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Abstract: The relations between the Faddeev functions and the functions of classical scattering theory are found in the complex domain at fixed energy. For the three-dimensional case (without assumption of "smallness" of the potential) it is proved that the exponentially decreasing potential is uniquely determined by its scattering amplitude at fixed energy.

Introduction

We consider the three-dimensional Schrödinger equation at fixed positive energy with a real exponentially decreasing potential,

$$-\Delta\psi + v(x)\psi = E\psi\,, \quad x \in \mathbb{R}^3\,, \quad E > 0\,. \tag{0.1}$$

The potential $v(x)$ is called exponentially decreasing if

$$v(x) \in L^\infty(\mathbb{R}^3) \quad \text{and} \quad \exists\alpha > 0\,,\ \exists\beta > 0 \quad \text{such that} \quad |v(x)| < \beta e^{-\alpha|x|}\,. \tag{0.2}$$

We consider the Faddeev functions $G(x,k)$, $\psi(x,k)$, $h(k,\ell)$,

$$G(x,k) = -\left(\frac{1}{2\pi}\right)^3 e^{ikx} \int\limits_{\xi\in\mathbb{R}^3} \frac{e^{i\xi x}d\xi}{\xi^2 + 2k\xi}\,, \tag{0.3}$$

$$\psi(x,k) = e^{ikx} + \int\limits_{y\in\mathbb{R}^3} G(x-y,k)v(y)\psi(y,k)dy\,, \tag{0.4}$$

$$h(k,\ell) = \left(\frac{1}{2\pi}\right)^3 \int\limits_{x\in\mathbb{R}^3} e^{-i\ell x}\psi(x,k)v(x)dx\,, \tag{0.5}$$

where k, $\ell \in \mathbb{C}^3$, $\ell^2 = k^2 = E$, $\operatorname{Im}\ell = \operatorname{Im}k \neq 0$.