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On "Hyperboloidal" Cauchy Data for Vacuum Einstein Equations and Obstructions to Smoothness of Scri

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Abstract: The relationship between the geometric properties of "hyperboloidal" Cauchy data for vacuum Einstein equations at the conformal boundary of the initial data surface and between the space-time geometry is analyzed in detail. We prove that a necessary condition for existence of a smooth or a polyhomogeneous Scri (i.e., a Scri around which the metric is expandable in terms of $r^{-j}\log^i r$ rather than in terms of r^{-j}) is the vanishing of the shear of the conformal boundary of the initial data surface. We derive the "boundary constraints" which have to be satisfied by an initial data set for compatibility with Friedrich's conformal framework. We show that a sufficient condition for existence of a smooth Scri (not necessarily complete) is the vanishing of the shear of the conformal boundary of the initial data surface and smoothness up to boundary of the conformally rescaled initial data. We also show that the occurrence of some log terms in an asymptotic expansion at the conformal boundary of solutions of the constraint equations is related to the non-vanishing of the Weyl tensor at the conformal boundary.

1. Introduction

In the papers of Bondi et al. [3], Sachs [18] and Penrose [17] which laid the foundations of our present understanding of the gravitational radiation, some rather strong hypotheses about the asymptotic behaviour of the gravitational field in lightlike directions were postulated. In a recent study by ourselves [1] (cf. also [2]) of the asymptotic properties of solutions of constraint equations on spacelike hypersurfaces intersecting "Scri" transversally it has, however, been observed that generic Cauchy data constructed in such a setting by the "conformal method" failed to be smoothly

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