Commun. Math. Phys. 161, 515–532 (1994)

Communications in Mathematical Physics

Perturbative Renormalization of Massless ϕ_4^4 with Flow Equations

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Received: 16 February 1993 / in revised form: 27 September 1993

Abstract: Perturbative renormalizability proofs in the Wilson-Polchinski renormalization group framework, based on flow equations, were so far restricted to massive theories. Here we extend them to Euclidean massless ϕ_4^4. As a by-product of the proof we obtain bounds on the singularity of the Green functions at exceptional momenta in terms of the exceptionality of the latter. These bounds seem to be new and are quite sharp.

1. Introduction

In recent years the authors have discussed the renormalization problem of perturbative field theory in a series of papers [1–5]. The method has been that of the renormalization group of Wilson as applied to perturbation theory in the form of differential flow equations by Polchinski [6]. We started by putting Polchinski's result on the renormalizability of massive ϕ_4^4 on a rigorous footing after simplifying the method of proof and included general (physical) renormalization conditions. The next step was to extend the method to QED [2] with a massive photon, where the main difficulty came from the fact that the regularization in the flow equation approach necessarily violates gauge invariance. (It is straightforward to convince oneself that the ϕ_4^4 proof works also for a general renormalizable (by power counting) massive Euclidean theory as long as there is no additional constraint, not respected by the regularization.) Furthermore we treated composite operator renormalization and the short distance expansion [3, 4], thus proving the method to be well-adapted also for more advanced and intricate issues in the field. Finally it turned out particularly suited for studying questions of convergence of the regularized theory to the renormalized one which go under the name of Symanzik's improvement programme [5].

Any method has, of course, its specific advantages and difficulties. In our framework we count among the latter that the regularization violates gauge invariance

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Supported by the Swiss National Science Foundation