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Approach to Equilibrium of Glauber Dynamics in the One Phase Region

II. The General Case

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Abstract: We develop a new method, based on renormalization group ideas (block decimation procedure), to prove, under an assumption of strong mixing in a finite cube Λ_0, a Logarithmic Sobolev Inequality for the Gibbs state of a discrete spin system. As a consequence we derive the hypercontractivity of the Markov semigroup of the associated Glauber dynamics and the exponential convergence to equilibrium in the uniform norm in all volumes Λ "multiples" of the cube Λ_0.

1. Preliminaries, Definitions, and Results

In this paper we analyze the problem of the approach to equilibrium for a general, not necessarily ferromagnetic, Glauber dynamics, i.e. a single spin flip stochastic dynamics reversible with respect to the Gibbs measure of a classical discrete spin system with finite range, translation invariant interaction. We prove that, if the Gibbs measure satisfies a *Strong Mixing Condition* on a large enough finite cube Λ_0, then the Glauber dynamics reaches the equilibrium exponentially fast in time in the *uniform norm*, in any finite or infinite volume Λ, provided that Λ is a "multiple" of the basic cube Λ_0. Such a result has already been proved in our previous papers [MO1, MO2] in the so-called "attractive case" by ad hoc methods. Here we prove the result in greater generality by proving a Logarithmic Sobolev Inequality for the Gibbs measure of the system. We refer to [MO2] for a general introduction to the problem of approach to equilibrium in the one phase region for Glauber dynamics; in particular in [MO2] one finds a critical discussion of the various *finite volume* mixing conditions for the Gibbs state and of the role played by the shape of the volumes involved when getting near to a line of first order phase transition. We also refer the reader to the beautiful series of papers by Zegarlinski [Z1, Z2, Z3] and Zegarlinski and Stroock [SZ1, SZ2, SZ3], where the theory of the Logarithmic Sobolev Inequality for Gibbs states was developed and its role in the proof of fast convergence to equilibrium of general, not

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