Approach to Equilibrium of Glauber Dynamics in the One Phase Region

I. The Attractive Case

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Abstract: Various finite volume mixing conditions in classical statistical mechanics are reviewed and critically analyzed. In particular some *finite size conditions* are discussed, together with their implications for the Gibbs measures and for the approach to equilibrium of Glauber dynamics in *arbitrarily large* volumes. It is shown that Dobrushin-Shlosman's theory of *complete analyticity* and its dynamical counterpart due to Stroock and Zegarlinski, cannot be applied, in general, to the whole one phase region since it requires mixing properties for regions of *arbitrary* shape. An alternative approach, based on previous ideas of Olivieri, and Picco, is developed, which allows to establish results on rapid approach to equilibrium deeply inside the one phase region. In particular, in the ferromagnetic case, we considerably improve some previous results by Holley and Aizenman and Holley. Our results are optimal in the sense that, for example, they show for the first time fast convergence of the dynamics *for any temperature* above the critical one for the d-dimensional Ising model with or without an external field. In part II we extensively consider the general case (not necessarily attractive) and we develop a new method, based on renormalizations group ideas and on an assumption of strong mixing in a finite cube, to prove hypercontractivity of the Markov semigroup of the Glauber dynamics.

0. Introduction

Recently many efforts have been devoted, with increasing interest, to analyze the precise connections between i) mixing properties of Gibbs measures for lattice spin systems (typically expressed in terms of rapid decay of the truncated correlations), and ii) the (properly defined) speed of approach towards equiilibrium of some associated spin flib Glauber type dynamics. We have in mind, in particular, the basic paper by Holley [H2], and the subsequent works by Aizenman and Holley [AH] and Stroock and Zegarlinski [SZ], where such connections were established, first for ferromagnetic