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Invariants of the Length Spectrum and Spectral Invariants of Planar Convex Domains

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Abstract: This paper is concerned with a conjecture of Guillemin and Melrose that the length spectrum of a strictly convex bounded domain together with the spectra of the linear Poincaré maps corresponding to the periodic broken geodesics in Ω determine uniquely the billiard ball map up to a symplectic conjugation. We consider continuous deformations of bounded domains Ω_s, $s \in [0, 1]$, with smooth boundaries and suppose that Ω_0 is strictly convex and that the length spectrum does not change along the deformation. We prove that Ω_s is strictly convex for any s along the deformation and that for different values of the parameter s the corresponding billiard ball maps are symplectically equivalent to each other on the union of the invariant KAM circles. We prove as well that the KAM circles and the restriction of the billiard ball map on them are spectral invariants of the Laplacian with Dirichlet (Neumann) boundary conditions for suitable deformations of strictly convex domains.

1. Introduction

This paper is concerned with certain length spectrum invariants of a strictly convex and bounded planar domain Ω with a smooth boundary $\partial\Omega$. The motivation for studying such invariants comes from the inverse spectral problem formulated by Kac [12]. It is known [10, 18], that the length spectrum $\mathscr{L}(\Omega)$ of Ω is encoded in the spectrum of the Laplace operator Δ in Ω with Dirichlet (Neumann) boundary conditions, and that $\mathscr{L}(\Omega)$ can be extracted from the spectrum of Δ by means of the Poisson formula at least for generic domains. In this connection, Guillemin and Melrose [9] formulated the conjecture that the length spectrum of Ω and the spectra of the linear Poincaré maps of the periodic broken geodesics of Ω form together a complete set of symplectic invariants for the corresponding billiard ball map B. As it was mentioned in [9], this conjecture seems to be a little optimistic and the local version of it is more hopeful.

The first result in this direction was obtained by Marvizi and Melrose [16] who described new length spectrum invariants of a strictly convex domain Ω, studying the

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