Commun Math. Phys 161, 125–156 (1994)

Communications in Mathematical Physics

The Hidden Group Structure of Quantum Groups: Strong Duality, Rigidity and Preferred Deformations

P. Bonneau[1], M. Flato[1], M. Gerstenhaber[2], G. Pinczon[1]

[1] Université de Bourgogne, Laboratoire de Physique Mathématique, B.P. 138, 21004 Dijon Cedex – France. e-mail: flato@satie.u-bourgogne.fr

[2] Department of Mathematics, University of Pennsylvania, Philadelphia, PA 19104-6395, U.S.A. e-mail: mgersten@mail.sas upenn.edu and murray@math.upenn.edu

Received:14 June 1993

Abstract: A notion of well-behaved Hopf algebra is introduced; reflexivity (for strong duality) between Hopf algebras of Drinfeld-type and their duals, algebras of coefficients of compact semi-simple groups, is proved. A hidden classical group structure is clearly indicated for all generic models of quantum groups. Moyal-product-like deformations are naturally found for all FRT-models on coefficients and C^∞-functions. Strong rigidity ($H^2_{bi} = \{0\}$) under deformations in the category of bialgebras is proved and consequences are deduced.

Introduction

There presently exist at least four models of quantum groups, introduced respectively by Drinfeld (D-model) [6], Jimbo (J-model) [16], Faddeev–Reshetikhin–Takhtajan (FRT-model) [9] and Woronowicz (W-model) [23]. We apologize for missing other models or authors. All these models are Hopf algebras that intend to be "deformations," in the following sense: they depend on a parameter, say q (or e^{it}), and when $q = 1$ (or $t = 0$), one finds a very classical and well known Hopf algebra, such as, e.g.: the enveloping algebra of a simple Lie algebra (D), the algebra of coefficients on an algebraic reductive group (FRT), an algebra of continuous functions on a compact group (W). It is often claimed that the classical limit of the J-model is the enveloping algebra of the corresponding simple Lie algebra; however this claim is not quite correct (see e.g. [5] or [3]): in fact, the classical limit of the J-model is an extension of $\mathcal{U}(g)$ by r parities ($r = \operatorname{rank} g$). As a matter of fact this should have been obvious even a priori, since all deformations of the (multiplicative structure of the) enveloping algebra of a simple algebra are trivial [8], while the J-model is a non-trivial deformation of its classical limit (see the end of this introduction). It is also often asserted that the D and FRT-models are mutually dual. Although this claim seems quite reasonable because generators of D-models can be found in the dual of FRT-models [9], a canonical duality $*$ such that $\mathrm{D}^* = \mathrm{FRT}$ and $\mathrm{FRT}^* = \mathrm{D}$ has not yet been constructed.

Here is a short summary of some puzzling problems concerning quantum groups: