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Asymptotics of Heavy Atoms in High Magnetic Fields: II. Semiclassical Regions

Elliott H. Lieb[1,2,*], **Jan Philip Solovej**[2,**], **Jakob Yngvason**[3,***]

[1] Department of Physics, Jadwin Hall, Princeton University, P O. Box 708, Princeton, NJ, 08544, USA

[2] Department of Mathematics, Fine Hall, Princeton University, Princeton NJ, 08544, USA

[3] Science Institute, University of Iceland, Dunhaga 3, IS-107 Reykjavik, Iceland

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Abstract: The ground state energy of an atom of nuclear charge Ze in a magnetic field B is exactly evaluated to leading order as $Z \to \infty$ in the following three regions: $B \ll Z^{4/3}$, $B \sim Z^{4/3}$ and $Z^{4/3} \ll B \ll Z^3$. In each case this is accomplished by a modified Thomas–Fermi (TF) type theory. We also analyze these TF theories in detail, one of their consequences being the nonintuitive fact that atoms are spherical (to leading order) despite the leading order change in energy due to the B field. This paper complements and completes our earlier analysis [1], which was primarily devoted to the regions $B \sim Z^3$ and $B \gg Z^3$ in which a semiclassical TF analysis is numerically and conceptually wrong. There are two main mathematical results in this paper, needed for the proof of the exactitude of the TF theories. One is a generalization of the Lieb–Thirring inequality for sums of eigenvalues to include magnetic fields. The second is a semiclassical asymptotic formula for sums of eigenvalues that is *uniform* in the field B.

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