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Lipshitz Continuity of Gap Boundaries for Hofstadter-like Spectra

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Abstract. We consider an effective Hamiltonian H representing the motion of a single-band-two-dimensional electron in a uniform magnetic field. Then H belongs to the rotation algebra, namely the algebra of continuous functions over a non-commutative 2-torus. We define a non-commutative analog of smooth functions by mean of elements of class $C^{l,n}$, where l and n characterize respectively the degree of differentiability with respect to the magnetic field and the torus variables. We show that if H is of class $C^{1,3+\varepsilon}$, the gap boundaries of the spectrum of H are Lipshitz continuous functions of the magnetic field at each point for which the gap is open.

1. Introduction

The motion of a single-band-two-dimensional electron in a uniform magnetic field can be represented by an effective Hamiltonian H in the tight binding representation [5]. Namely it acts on the lattice $\mathbb{Z}^2$ as a function of the so-called "magnetic translations" [25] U and V. These are two unitaries submitted to the following commutation rule:

$$UV = e^{2i\pi\alpha} VU\,, \tag{1}$$

where $\alpha = \phi/\phi_0$ is the ratio between the flux ϕ in the unit cell of the lattice and the flux quantum $\phi_0 = e/h$.

The abstract C^*-algebra $\mathcal{A}_\alpha$ generated by two such unitaries was introduced by Rieffel [22] and subsequently Connes [9] showed that it has a differential structure which makes it a non-commutative smooth manifold. Moreover, given an interval I in the real line, the field $\alpha \in I \mapsto \mathcal{A}_\alpha$ is a continuous field of C^*-algebras [10, 23] and we will denote by $\mathcal{A}_I$ the C^*-algebra it generates.

One of the most famous examples of Hamiltonian built in this way is the so-called "Harper model" [12] given by:

$$H_{\text{Harper}} = U + U^* + V + V^*, \tag{2}$$

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