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Constrained Quantisation, Gauge Fixing and the Gribov Ambiguity

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Abstract: The role played by gauge fixing in the description of superselection sectors for a simple quantum mechanical system is analysed. By viewing this as a theory with constraints, it is shown that the possibility of having inequivalent gauge fixing conditions (Gribov's ambiguity) signals the existence of inequivalent reductions to a physical quantum theory, and hence superselection sectors. This point of view is contrasted with the more traditional one that identifies superselection sectors with inequivalent quantisations. It is argued that emphasising the role of gauge fixing (along with the Gribov problem) will allow for a more direct extension of these ideas to quantum field theory and, in particular, gauge theories.

1. Introduction

Classically a Yang–Mills theory is characterised by its coupling constant. After quantising, though, an additional parameter is needed, measuring the vacuum angle. We can think of this angle θ as labelling various superselection sectors; it then being a matter of experimentation (through the analysis of CP-violating effects) to fix its physical value. Initially θ arose from a semi-classical analysis [1]. However it is now clear (as discussed, for example, in [2]) that the emergence of such an angle reflects a general property of quantising when the configuration space[1] of the system has non-contractible loops (the θ angle then emerging from the representation theory of the first homotopy group, π_1, of this space).

The existence of such non-trivial loops in Yang–Mills theory follows from the fact that the group of all gauge transformations (in 3-dimensional space) is disconnected, and indeed has components labelled by the integers. Thus it is the set $\hat{\mathbb{Z}}$, of all irreducible unitary representations of the group of integers $\mathbb{Z}$, that determines the superselection sector: as is well known, $\hat{\mathbb{Z}} \simeq SO(2)$ – hence we get an angle.

[1] The configuration space in question being the space of physically inequivalent potentials $\mathscr{A}/\mathscr{G}$ (with $\mathscr{G}$ the group of gauge transformation) as opposed to the (extended configuration) space $\mathscr{A}$ of all Yang–Mills potentials – which is topologically quite trivial