On Foundation of the Generalized Nambu Mechanics

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Abstract: We outline basic principles of a canonical formalism for the Nambu mechanics – a generalization of Hamiltonian mechanics proposed by Yoichiro Nambu in 1973. It is based on the notion of a Nambu bracket, which generalizes the Poisson bracket – a "binary" operation on classical observables on the phase space – to the "multiple" operation of higher order $n \geq 3$. Nambu dynamics is described by the phase flow given by Nambu–Hamilton equations of motion – a system of ODE's which involves $n - 1$ "Hamiltonians." We introduce the fundamental identity for the Nambu bracket – a generalization of the Jacobi identity – as a consistency condition for the dynamics. We show that Nambu bracket structure defines a hierarchy of infinite families of "subordinated" structures of lower order, including Poisson bracket structure, which satisfy certain matching conditions. The notion of Nambu bracket enables us to define Nambu–Poisson manifolds – phase spaces for the Nambu mechanics, which turn out to be more "rigid" than Poisson manifolds – phase spaces for the Hamiltonian mechanics. We introduce the analog of the action form and the action principle for the Nambu mechanics. In its formulation, dynamics of loops ($n - 2$-dimensional chains for the general n-ary case) naturally appears. We discuss several approaches to the quantization of Nambu mechanics, based on the deformation theory, path integral formulation and on Nambu–Heisenberg "commutation" relations. In the latter formalism we present an explicit representation of the Nambu–Heisenberg relation in the $n = 3$ case. We emphasize the role ternary and higher order algebraic operations and mathematical structures related to them play in passing from Hamilton's to Nambu's dynamical picture.

1. Introduction

In 1973 Nambu proposed a profound generalization of classical Hamiltonian mechanics [1]. In his formulation a triple (or, more generally, n-tuple) of "canonical" variables replaces a canonically conjugated pair in the Hamiltonian formalism and ternary (or, more generally, n-ary) operation – the Nambu bracket – replaces the usual Poisson bracket. Dynamics, according to Nambu, is determined by