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On Polymer Conformations in Elongational Flows

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Abstract: We consider various models of polymer conformations using paths of Gaussian processes such as Brownian motion. In each case, the calculation of the law of the moment of inertia of a random polymer structure (which is equivalent to the calculation of the partition function) is reduced to the problem of finding the law of a certain quadratic functional of a Gaussian process. We present a new method for computing the Laplace transforms of these quadratic functionals which exploit their special form via the Ray–Knight Theorem and which does not involve the classical method of eigenvalue expansions. We apply the method to several simple examples, then show how the same techniques can be applied to more complicated cases with the aid of a little excursion theory.

1. Introduction

This study extends and greatly simplifies previous work on polymer conformations in pure straining motions (or elongational flows). The main method used is a technique for characterizing the law of a quadratic functional of Brownian motion (or other Gaussian process) which also extends and greatly simplifies previous work in this area. Among other examples we consider the conformations of single-chain polymers with one end attached to a suspended particle and ring polymers, in an elongational flow at zero Reynolds number, that is, in a limit where inertia forces are neglected. In each case we compute the law of the moment of inertia of the random polymer. The laws of other functionals can also be computed, but we concentrate on the moment of inertia as a particularly important physical quantity. To understand why we can convert the polymer problem into one on quadratic functionals of Gaussian processes, we first present some of the physics of the problem.

An elongational (or straining) velocity field $u(x)$ is described by

$$u(x) = Ex\,, \tag{1.1}$$

where x is the position vector and E is a traceless symmetric matrix called the rate-of-strain tensor. The force on a single rigid particle in a zero Reynolds number