Commun Math Phys 160, 49–80 (1994)

Communications in
Mathematical Physics
© Springer-Verlag 1994

Hydrodynamic Limit of the Stationary Boltzmann Equation in a Slab

R. Esposito[1], **J.L. Lebowitz**[2], **R. Marra**[3]

[1] Dipartimento di Matematica, Università di Roma Tor Vergata, I-00133 Roma, Italy
[2] Mathematics and Physics Departments, Rutgers University, New Brunswick, NJ 08903, USA
[3] Dipartimento di Fisica, Università di Roma Tor Vergata, I-00133 Roma, Italy

Received: 20 March 1993

Abstract: We study the stationary solution of the Boltzmann equation in a slab with a constant external force parallel to the boundary and complete accommodation condition on the walls at a specified temperature. We prove that when the force is sufficiently small there exists a solution which converges, in the hydrodynamic limit, to a local Maxwellian with parameters given by the stationary solution of the corresponding compressible Navier–Stokes equations with no-slip boundary conditions. Corrections to this Maxwellian are obtained in powers of the Knudsen number with a controlled remainder.

1. Introduction

In this paper we continue our study of the derivation of hydrodynamic equations from the Boltzmann equation (BE), a problem which goes back to Hilbert [1]. The BE is believed to accurately describe the time evolution of rarefied gases on a "kinetic" scale intermediate between the microscopic and macroscopic [2]. To go from the BE to the macroscopic (hydrodynamic) descriptions the locally conserved density fields have to be slowly varying on the kinetic (to which we shall refer from now on as microscopic) scale but have sensible space variations over macroscopic distances. Let ε be the ratio between microscopic and macroscopic space units (usually called the Knudsen number). It can be shown that the conserved densities, *observed at microscopic times of order* ε^{-1}, converge, as $\varepsilon \to 0$, to macroscopic fields whose time evolution is given by the solution of the Euler equations (EE) (at least when the latter have a smooth solution) [3, 4, 5]. This derivation of the EE in the above hydrodynamical (Euler) scaling limit is consistent with (indeed made possible by) the fact that the EE are themselves invariant under uniform space and time scaling.

Unfortunately there is no such scale invariance (and thus no such scaling limit) for the Navier–Stokes equations (NSE). The NSE are usually deduced, via the Chapman–Enskog expansion (see [6]), as corrections to the EE on the Euler time scale ε^{-1} with viscosity coefficient and thermal conductivity of order ε. To describe situations which discriminate between the two equations when the Knudsen