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On the Spectra of Schrödinger Operators

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Abstract: We give two formulas for the lowest point $\mathcal{T}$ in the spectrum of the Schrödinger operator $L = -(d/dt)p(d/dt) + q$, where the coefficients p and q are real-valued, bounded, uniformly continuous functions on the real line. We determine whether or not $\mathcal{T}$ is an eigenvalue for L in terms of a set of probability measures on the maximal ideal space of the C^*-algebra generated by the translations of p and q.

Introduction

In this paper, we will study the Schrödinger operator

$$L = -\left(\frac{d}{dt}\right)p\left(\frac{d}{dt}\right) + q$$

on $\mathcal{D}_2 \subset L^2(\mathbf{R})$. As usual, the domain $\mathcal{D}_2$ of this operator is the collection of functions $f \in L^2(\mathbf{R})$ which have the property that f and f' are absolutely continuous functions on every finite interval and $f', f'' \in L^2(\mathbf{R})$. We assume that p and q are real-valued, bounded, uniformly continuous functions on $\mathbf{R}$. In addition, we assume that p' is also a bounded, uniformly continuous function on $\mathbf{R}$ and that there is a $c > 0$ such that $p(t) \geq c$ for every $t \in \mathbf{R}$. It is well known that, under these assumptions, L is a self-adjoint operator on $\mathcal{D}_2$. The main goal of this paper is to study the lowest point $\mathcal{T} = \inf\{\lambda : \lambda \in \sigma(L)\}$ of the spectrum of L. There have been estimates of the value $\mathcal{T}$ in the literature when the coefficients p and q of the operator have recurrence properties [4]. We will give two formulas for the value $\mathcal{T}$. These formulas are related to a C^*-algebra associated with the functions p and q.

Before we state our results, some definitions are necessary. For a function f defined on $\mathbf{R}$, by a translation of f we mean a function f_s given by the formula $f_s(t) = f(t+s)$. We denote by $\mathcal{A}$ the C^*-algebra generated by all the translations of p, p', q and all

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