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Stark Ladders of Resonances: Wannier Ladders and Perturbation Theory

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Abstract: Let H_B be any fixed one-dimensional Bloch Hamiltonian with only the first m gaps open and $H_F = H_B + Fx$ be the corresponding Stark Hamiltonian. For any positive F small enough H_F has only m ladders of sharp resonances given by the analytic translation method, the decoupled band approximation and the regular perturbation theory. This way, the Wannier conjecture becomes a definite regular perturbation theory for the Stark ladders as eigenvalues of the translated Hamiltonian.

I. Introduction

In 1960 Wannier [23] suggested the existence of (Stark) ladders of bound states (or resonances) in the Bloch Stark Hamiltonian:

$$H_F = -\hbar^2 \frac{d^2}{dx^2} + V(x) + Fx\,, \qquad V(x) = V(x+a) \quad \text{and} \quad F, a > 0\,, \tag{I.1}$$

which does not have any bound state for zero external electric field. This location effect given by the external uniform field on Bloch problems is in agreement with the tilted bands picture of Zener. Actually Wannier proved that the one band approximation gives a (Wannier) ladder of bound states for each finite band, but he was not able to extend the result to the full problem. Even in the absence of definite experimental results, the existence of the ladders was put in doubt [24]. Some years ago this attitude changed because of the accurate numerical works [4, 6], the new experimental results [3] and a rigorous definition of ladder resonances by means of eigenvalues of a suitable operator [15]. In 1982 Avron [2], assuming the existence of ladders of resonances and using the crystal momentum representation, studied the width behavior of the resonances in the Fermi Golden Rule approximation. Recently, rigorous proofs of existence have appeared in different regimes: for large electric field strength [1], for electric field strength small with $\hbar$ [8,10] and for large period [13].

We want now to continue and to extend the line of research started with Wannier in order to obtain the existence of resonances for a fixed Bloch model and weak