Commun Math Phys 159, 581–591 (1994)

Communications in
Mathematical Physics

On the Parametrisation of Unitary Matrices by the Moduli of their Elements

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Received December 18, 1992

Abstract: The parametrisation of an $n \times n$ unitary matrix by the moduli of its elements is not a well posed problem, i.e. there are continuous and discrete ambiguities which naturally appear. We show that the continuous ambiguity is $(n-1)(n-3)$-dimensional in the general case and $\frac{n(n-3)}{2}$-dimensional in the symmetric case $S_{ij} = S_{ji}$. We give also lower bounds on the number of discrete ambiguities, the number of solutions being at least $2^{\frac{n(n-3)}{2}}$ in the first case and $2^{\left[\frac{n}{2}\right]\left[\frac{n-1}{2}\right]-1}$ for the symmetric one, where $[r]$ denotes the integral part of r.

1. Introduction

There has been much recent interest in the problem of reconstructing the phases of a unitary matrix from the knowledge of the moduli of its matrix elements [1–2, 4–5, 7–8]. Stated in this general form the problem is of broad interest for people working in circuit theory, phase shift analyses, multichannel scattering, standard model, CP violation, etc.

Actually the last two items explicitly raised the problem alluded to in the title. People working in the study of the Cabibbo-Kobayashi-Maskawa (CKM) mass matrix had to take into account the experimental fact that almost all the accessible information we have about the unitary CKM mass matrix is given in terms of the moduli of its matrix elements.

From a pragmatic point of view a parametrisation of a unitary matrix by the moduli of its elements is very appealing. On the other hand, such a parametrisation is not natural. A natural one would be one whose parameters are free, i.e. there are no supplementary constraints upon them to enforce unitarity. Natural parametrisations are the Euler-type parametrisation given by Murnagham [10], or that found by us [6] which generalises to an arbitrary dimension the one given by Watson [11] for 2×2 unitary matrices.