Commun Math Phys 159, 549–579 (1994)

Communications in
Mathematical
Physics

Chern-Simons States at Genus One

Fernando Falceto[1], **Krzystof Gawedzki**[2]

[1] Depto. Física Teórica, U. Zaragoza, E-50009 Zaragoza, Spain
[2] C.N.R.S., I.H.E.S., F-91440 Bures-sur-Yvette, France

Received: 3 November 1992/in revised form: 15 July 1993

Abstract: We present a rigorous analysis of the Schrödinger picture quantization for the $SU(2)$ Chern-Simons theory on 3-manifold torus $\times$ line, with insertions of Wilson lines. The quantum states, defined as gauge covariant holomorphic functionals of $su(2)$-connections on the torus, are expressed by degree $2k$ theta-functions satisfying additional conditions. The conditions are obtained by splitting the space of semistable $su(2)$-connections into nine submanifolds and by analyzing the behavior of states at four codimension 1 strata. We construct the Knizhnik-Zamolodchikov-Bernard connection allowing to compare the states for different complex structures of the torus and different positions of the Wilson lines. By letting two Wilson lines come together, we prove a recursion relation for the dimensions of the spaces of states which, together with the (unproven) absence of states for spins $> \frac{1}{2}$ level implies the Verlinde dimension formula.

1. Introduction

Since the Chern-Simons (CS) theory was revisited by Witten in [33], with the stress on its topological nature, a considerable effort has been made to study different aspects of the theory. From the point of view of covariant quantization, the CS model was used to obtain 3-manifold, knot and link invariants, either by surgery [33, 22] or in the perturbative expansion [8, 19, 4].

Here we will discuss a complementary aspect of the CS theory: its canonical quantization. It has been argued in [33] that the space of Schrödinger states of the CS theory with a compact Lie group G, in the presence of Wilson lines, is isomorphic to the space of conformal blocks of the associated group G Wess-Zumino-Witten (WZW) conformal field theory. The Wilson lines correspond to the insertions of the primary fields in the two-dimensional model. The conformal blocks are holomorphic sections of the Friedan-Shenker (FS) [14] vector bundle over the moduli space of complex structures of a punctured Riemann surface. By definition, they are horizontal with respect to a flat projective connection. In the WZW model, the latter is a generalization