On Geometrical Interpretation
of the p-Adic Maslov Index

E. I. Zelenov[*]

Steklov Mathematical Institute, Vavilov Str 42, GSP-1, 117966, Moscow, Russia

Abstract: A set of selfdual lattices Λ in a two-dimensional p-adic symplectic space $(\mathscr{V}, \mathscr{B})$ is provided by an integer valued metric d. A realization of the metric space (Λ, d) as a graph Γ is suggested and this graph has been linked to the Bruhat-Tits tree. An action of symplectic group $\mathrm{Sp}(\mathscr{V})$ on a set of cycles of length three of the graph Γ is considered and a geometrical interpretation of the p-adic Maslov index is given in terms of this action.

Introduction

In the paper [Z] a definition of the p-adic Maslov index of a triple of selfdual lattices in a two-dimensional p-adic symplectic space $(\mathscr{V}, \mathscr{B})$ was suggested. In general the construction is as follows. For any selfdual lattice $\mathscr{L}$ in $(\mathscr{V}, \mathscr{B})$ we define an irreducible unitary representation $(H(\mathscr{L}), W_{\mathscr{L}})$ of the Heisenberg group $\tilde{\mathscr{V}}$ of space $(\mathscr{V}, \mathscr{B})$ in a separable Hilbert space $H(\mathscr{L})$. These representations are unitary equivalent and hence for any pair $(H(\mathscr{L}_1), W_{\mathscr{L}_1})$, $(H(\mathscr{L}_2), W_{\mathscr{L}_2})$ of two such representations there exists an intertwining operator $F_{\mathscr{L}_2, \mathscr{L}_1} : H(\mathscr{L}_1) \to H(\mathscr{L}_2)$. Therefore for any triple of such representations the operator $F = F_{\mathscr{L}_1, \mathscr{L}_3} F_{\mathscr{L}_3, \mathscr{L}_2} F_{\mathscr{L}_2, \mathscr{L}_1}$ commutes with all operators $W_{\mathscr{L}_1}(x)$, $x \in \tilde{\mathscr{V}}$. Thus F is proportional to an identity operator $\mathrm{Id} : F = \mathfrak{m}(\mathscr{L}_1, \mathscr{L}_2, \mathscr{L}_3) \, \mathrm{Id}$. The complex number $\mathfrak{m}(\mathscr{L}_1, \mathscr{L}_2, \mathscr{L}_3)$ is the *p-adic Maslov index* of a triple $(\mathscr{L}_1, \mathscr{L}_2, \mathscr{L}_3)$ of selfdual lattices. In the paper [Z] simple properties of this index and explicit formulas for the index are given.

This paper is devoted to a geometrical interpretation of the p-adic Maslov index (we suppose that $p \neq 2$). This interpretation is given in terms of an action of p-adic symplectic group $\mathrm{Sp}(\mathscr{V})$ on a space Λ of selfdual lattices. Section 2 is concerned with the space Λ of selfdual lattices in a two-dimensional symplectic space $(\mathscr{V}, \mathscr{B})$ over the field $\mathbb{Q}_p$ of p-adic numbers. It turns out that the space Λ can be provided with an

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[*] e-mail: zelenov@mph.mian.su