Commun. Math. Phys. 159, 503–537 (1994)

Communications in Mathematical Physics


# New Quasi-Exactly Solvable Hamiltonians in Two Dimensions

**Artemio González-López[1]*, Niky Kamran[2]**, Peter J. Olver[3,]***,******

[1] Departamento de Física Teórica II, Universidad Complutense, 28040 Madrid, Spain
[2] Department of Mathematics, McGill University, Montréal, Québec, Canada H3A 2K6
[3] School of Mathematics, University of Minnesota, Minneapolis, Minnesota 55455, USA

Received: 8 March 1993

**Abstract:** Quasi-exactly solvable Schrödinger operators have the remarkable property that a part of their spectrum can be computed by algebraic methods. Such operators lie in the enveloping algebra of a finite-dimensional Lie algebra of first order differential operators – the "hidden symmetry algebra." In this paper we develop some general techniques for constructing quasi-exactly solvable operators. Our methods are applied to provide a wide variety of new explicit two-dimensional examples (on both flat and curved spaces) of quasi-exactly solvable Hamiltonians, corresponding to both semisimple and more general classes of Lie algebras.

## 1. Introduction

The spectral problems of non-relativistic quantum mechanics fall within two general categories. In the first category, we have the small number of so-called exactly solvable problems, that is Schrödinger operators whose entire spectrum can be determined by algebraic methods. The simplest example of such a problem is given by the harmonic oscillator. In the second category, we have the Schrödinger operators whose complete spectrum cannot be computed exactly, but only approximated numerically at the very best.

Over the past decade, there has been a fair amount of interest in trying to construct physically significant systems which may not be exactly solvable, but for which part of the spectrum can be computed exactly by algebraic methods. In the early 1980's, Alhassid, Gürsey, Iachello, Levine and collaborators, [1, 3, 12] introduced the concept of a "spectrum generating algebra" to construct models for complicated molecules whose point spectrum could be analyzed algebraically.

* Supported in Part by DGICYT Grant PS 89-0011.
** Supported in Part by an NSERC Grant.
*** Supported in Part by NSF Grant DMS 92-04192.
**** On leave from Department of Mathematics, University of Maryland, College Park, MD 20742, U.S.A.