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On the Integrated Density of States for Crystals with Randomly Distributed Impurities

Rainer Hempel[1]*, **Werner Kirsch**[2]

[1] Mathematisches Institut der Universität München, Theresienstr. 39, D-80333 München, Germany
[2] Institut für Mathematik und Sonderforschungsbereich 237, Ruhr-Universität Bochum, D-44780 Bochum, Germany

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Abstract: In the present paper, we discuss spectral properties of a periodic Schrödinger operator which is perturbed by randomly distributed impurities; such operators occur as simple models for crystals (or semi-conductors) with impurities. While the spectrum itself is independent of the concentration p of impurities, for $0 < p < 1$, we focus our attention on the limiting behavior of the integrated density of states ρ_p of the random Schrödinger operator, inside a spectral gap of the periodic operator, as $p \to 0$. Denoting by U_0 the set of eigenvalues (in the gap) of the reference problem having precisely one impurity (located at the origin, say), we show that the integrated density of states concentrates around the points of U_0, in the sense that $\rho_p(U_\varepsilon)$ is of order p, for any fixed ε-neighborhood U_ε of U_0, while $\rho_p(K) \leqq C \cdot p^2$, for any compact subset K of the gap which does not intersect U_ε.

1. Introduction

We consider a simple model for a crystalline solid with impurities. In this model, atoms of a pure crystal are replaced by atoms of a different species (impurities) in a random way (i.e., at a lattice site there is an impurity with probability p independent of the other sites). The spectrum Σ of the resulting alloy can be described rather explicitly (see e.g. [KM1, EK, K1]). For any subset I of the lattice $\mathbb{Z}^\nu$ let us denote by Σ_I the spectrum of the crystal with impurities (exactly) at the sites $i \in I$. Then Σ, the spectrum of the random alloy, is (almost surely) given by:

$$\Sigma = \overline{\bigcup \Sigma_I}\,, \tag{1.1}$$

where the union is extended over all *finite* subsets I of $\mathbb{Z}^\nu$. Thus, Σ consists of the spectrum of the pure crystal plus (the closure of) all the eigenvalues of systems with finitely many impurities. The closure of these eigenvalues will form, as a rule, bands

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