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On the Distribution of Zeros of a Ruelle Zeta-Function

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Abstract: We study the limit distribution of zeros of a Ruelle ζ-function for the dynamical system $z \mapsto z^2 + c$ when c is real and $c \to -2-0$ and apply the results to the correlation functions of this dynamical system.

Consider the dynamical system defined by the complex polynomial map f_c: $z \mapsto z^2 + c$, where $c < -2$. We use the notions and results of the iteration theory of rational functions (see for example [5]). Denote by f_c^{*n} the n^{th} iterate of the function f_c. The Julia set $J(f_c)$ is a Cantor set on the real line. So in particular all finite periodic points are real. This system is expanding (hyperbolic) on its Julia set. When $c = -2$ the Julia set is the segment $[-2, 2]$ and the map $P = f_{-2}$ is not expanding anymore. We have the conjugation

$$P \circ \phi = \phi \circ Q \,, \tag{1}$$

where ϕ: $[0, 1] \to [-2, 2]$, $t \mapsto 2\cos \pi t$ and

$$Q = \begin{cases} t \mapsto 2t & 0 \leqq t \leqq 1/2\,, \\ t \mapsto 2-2t, & 1/2 \leqq t \leqq 1\,. \end{cases}$$

Remark that the chaotic dynamic of P on $[-2,2]$ was investigated by J. von Neuman and S. Ulam on one of the first computers.

We are going to study the dynamics of f_c, $c < -2$ when $c \to -2$ and then compare it with the behavior of the limit system P. The chaotic dynamics of f_c has to be described in probabilistic terms. This can be done by introducing an appropriate invariant probability measure σ_c on the Julia set. We will show that the rate of asymptotic decrease of correlation functions of the system (f_c, v_c) changes dramatically when we pass to the limit system as $c \to -2$.

Our tool is the Thermodynamic Formalism [12–15]. Let us introduce the main objects of this theory in our particular case. Consider the Fréchet space $C^\infty(U)$ of

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