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# Global Stability of Large Solutions to the 3D Navier-Stokes Equations

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**Abstract:** We prove the stability of mildly decaying global strong solutions to the Navier-Stokes equations in three space dimensions. Combined with previous results on the global existence of large solutions with various symmetries, this gives the first global existence theorem for large solutions with approximately symmetric initial data. The stability of unforced 2D flow under 3D perturbations is also obtained.

## 1. Introduction

It is well known that there are always global weak solutions to the three dimensional Navier-Stokes system

$$u_t - \nu \Delta u + (u \cdot \nabla)u + \nabla p = f\,, \qquad \text{in } [0,\infty) \times \Omega\,, \tag{1.1}$$

$$\nabla \cdot u = 0\,, \qquad \text{in } [0,\infty) \times \Omega\,, \tag{1.2}$$

$$u = 0\,, \qquad \text{on } [0,\infty) \times \partial\Omega\,, \tag{1.3}$$

$$u(0,\cdot) = u_0\,, \qquad \text{in } \Omega\,. \tag{1.4}$$

Here, as usual, $u = u(t, x_1, x_2, x_3) = u(t,x) = (u_1, u_2, u_3)$ is the velocity vector, $\nu > 0$ is the viscosity, $p = p(t,x)$ is the pressure, $f = f(t,x)$ is the external force, and $u_0(x)$ is the initial velocity. Equations (1.1)–(1.4) describe the motion of a viscous, incompressible fluid in a domain $\Omega \subset \mathbb{R}^3$. Under minimal assumptions on the data $(u_0, f)$, the existence of a weak solution is guaranteed by the results of Leray [9] and Hopf [5]. The uniqueness of $(u,p)$, up to an additive constant for the pressure $p$, remains open in general and is only known for strong solutions which *a priori* exist locally. The global existence of small strong solutions has been proved, but for large data, strong global solutions are known to exist only under the assumption of certain spatial symmetries.