

# The Algebra of the Energy-Momentum Tensor and the Noether Currents in Classical Non-Linear Sigma Models

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**Abstract:** The recently derived current algebra of classical non-linear sigma models on arbitrary Riemannian manifolds is extended to include the energy-momentum tensor. It is found that in two dimensions the energy-momentum tensor  $\theta_{\mu\nu}$ , the Noether current  $j_\mu$  associated with the global symmetry of the theory and the composite field  $j$  appearing as the coefficient of the Schwinger term in the current algebra, together with the derivatives of  $j_\mu$  and  $j$ , generate a closed algebra. The subalgebra generated by the light-cone components of the energy-momentum tensor consists of two commuting copies of the Virasoro algebra, with central charge  $c = 0$ , reflecting the classical conformal invariance of the theory, but the current algebra part and the semidirect product structure are quite different from the usual Kac-Moody/Sugawara type construction.

In a recent paper [1], we have derived the current algebra for classical non-linear sigma models defined on Riemannian manifolds. This algebra is quite simple to write down and yet does not seem to belong to any of the algebras which are well known in mathematical physics, mainly because it involves non-standard (in particular, non-central) extensions of loop algebras [4].

On the other hand, the classical non-linear sigma model in two dimensions is conformally invariant, so its energy-momentum tensor must satisfy the classical version of the standard commutation relations of conformal field theory, that is, under Poisson brackets its light-cone components must generate two commuting copies of the Witt algebra (the Virasoro algebra with vanishing central charge). We shall verify that this is indeed the case. Moreover, we shall derive the Poisson bracket relations between the energy-momentum tensor on the one hand and the Noether currents on the other hand. The resulting total algebra exhibits, in a concrete field-theoretical model with continuous internal symmetries, the possibility of reconciling conformal

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