Commun. Math. Phys. 159, 287–318 (1994)

Communications in
Mathematical
Physics

Stability of Moving Fronts in the Ginzburg–Landau Equation

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Received: 13 January 1993/in revised form: 24 March 1993

Abstract: We use Renormalization Group ideas to study stability of moving fronts in the Ginzburg–Landau equation in one spatial dimension. In particular, we prove stability of the real fronts under complex perturbations. This extends the results of Aronson and Weinberger to situations where the maximum principle is inapplicable and constitutes a step in proving the general marginal stability hypothesis for the Ginzburg–Landau equation.

1. Introduction

There are very few general approaches to the study of long time existence and asymptotics of solutions of nonlinear parabolic partial differential equations. Typically one has to resort to the use of positivity properties of the linear semigroup, e.g. the use of maximum principle and then use comparison theorems together with compactness arguments to obtain the asymptotics. Such approaches usually work only for equations of special form, e.g. with second order linear part, and positive initial data.

It was noted in [3, 12, 13], that scaling and renormalization group (RG) concepts that were very successful in statistical mechanics and quantum field theory are also applicable to this study. In [5, 6] we have been developing a mathematical RG theory to prove global existence and detailed long time asymptotics for classes of nonlinear parabolic equations. The RG approach does not depend on the applicability of positivity conditions such as the maximum principle. The theory moreover shows how *universality* emerges in such equations: the long time asymptotics is independent on the initial data and the equation within classes of data and equations.

* Supported by EC grant SC1-CT91-0695
** Supported by NSF grant DMS-8903041