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Batalin–Vilkovisky Algebras and Two-Dimensional Topological Field Theories

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Abstract: By a Batalin–Vilkovisky algebra, we mean a graded commutative algebra A, together with an operator $\Delta\colon A_\cdot \to A_{\cdot+1}$ such that $\Delta^2 = 0$, and $[\Delta, a] - \Delta a$ is a graded derivation of A for all $a \in A$. In this article, we show that there is a natural structure of a Batalin–Vilkovisky algebra on the cohomology of a topological conformal field theory in two dimensions. We make use of a technique from algebraic topology: the theory of operads.

Batalin–Vilkovisky algebras are a new type of algebraic structure on graded vector spaces, which first arose in the work of Batalin and Vilkovisky on gauge fixing in quantum field theory: a Batalin–Vilkovisky algebra is a differential graded commutative algebra together with an operator $\Delta\colon A_\cdot \to A_{\cdot+1}$ such that $\Delta^2 = 0$, and

$$\begin{aligned}\Delta(abc) = {} & \Delta(ab)c + (-1)^{|a|}a\Delta(bc) + (-1)^{(|a|-1)|b|}b\Delta(ac) \\ & - (\Delta a)bc - (-1)^{|a|}a(\Delta b)c - (-1)^{|a|+|b|}ab(\Delta c)\,.\end{aligned}$$

(Some references for Batalin–Vilkovisky algebras, with diverse applications to physics, are Schwarz [20], Witten [22] and Zwiebach [24].)

In this article, we show that there is a natural structure of a Batalin–Vilkovisky algebra on the cohomology of a topological conformal field theory in two dimensions. Lian and Zuckerman [16] have constructed a Batalin–Vilkovisky structure on the cohomology of a topological chiral field theory, and calculated it explicitly in the case of $D = 2$ string theory. Our approach to the study of this structure is quite different from that of Lian and Zuckerman, and makes use of a technique from algebraic topology: the theory of operads. (It was lectures of Maxim Kontsevich at Harvard which first suggested a role for operads in topological field theory.) As a result, we obtain the stronger result that no additional relations on the

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