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Large Deviations for Young Measures and Statistical Mechanics of Infinite Dimensional Dynamical Systems with Conservation Law

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Abstract: We describe an approach, based on Baldi's large deviation theorem, to carry out the statistical mechanics of a class of infinite dimensional dynamical systems.

I. Introduction

In previous works [32–34] we showed that large deviation theory (Sanov's theorem), used in a Young measure framework, provides an efficient tool to carry out the thermodynamic limit yielding the equilibrium states for two-dimensional Euler equations. The interest of this approach was reinforced by the confirmation both experimentally [10, 38] and by numerical simulations [40] of the theoretical predictions in the case of a vortex patch.

One aim of this paper is to provide a complete proof of the large deviation estimates used in [33]. Our proof is an application of Baldi's large deviation theorem [3] (in a slightly modified version). More generally, we show that Baldi's theorem is an elegant and powerful tool to carry out thermodynamic limits in various functional frameworks.

Another aim is to describe a class of infinite dimensional dynamical systems to which the theory can be applied.

It is out of the scope of this paper to review the main contributions to statistical hydrodynamics, nevertheless some comments and references are given in Sect. IV. It is also worth noticing that a work close to ours, although expounded from a more physical point of view, is developed by Miller et al. [26].

II. Baldi's Large Deviation Theorem and Thermodynamic Limits

Baldi's theorem gives general conditions under which a family of probability measures on a locally convex topological vector space has the large deviation property.

As we will see, it provides a powerful tool to carry out thermodynamic limits for infinite dimensional systems.