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Composition of Kinetic Momenta: The $\mathcal{U}_q(sl(2))$ Case

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Abstract. The tensor products of (restricted and unrestricted) finite dimensional irreducible representations of $\mathcal{U}_q(sl(2))$ are considered for q a root of unity. They are decomposed into direct sums of irreducible and/or indecomposable representations.

1. Introduction

When the parameter of deformation q is not a root of unity, the theory of representations of quantum algebras $\mathcal{U}_q(\mathcal{G})$ (with $\mathcal{G}$ a semi-simple Lie algebra) is equivalent to the classical theory [1]. In the following, we consider $\mathcal{U}_q(sl(2))$, with q a root of unity. In this case, the dimension of the finite dimensional irreducible representations (irreps) is bounded, and a new type of representations occurs, depending on continuous parameters [2–5]. Moreover, finite dimensional representations are not always direct sums of irreps: they can contain indecomposable sub-representations. Some kinds of indecomposable representations actually appear in the decomposition of tensor products of irreps.

Another peculiarity with q a root of unity is that the fusion rules are generally not commutative. There exist, however, many sub-fusion-rings that are commutative. The well-known one is the fusion ring generated by the irreps of the finite dimensional quotient of $\mathcal{U}_q(sl(2))$ [6–8)]. Families of larger commutative fusion ring that contain the latter will also be defined later.

The following section is devoted to definitions, to the description of the centre of $\mathcal{U}_q(sl(2))$, and finally recalls the classification of the irreps of $\mathcal{U}_q(sl(2))$. The irreps of $\mathcal{U}_q(sl(2))$ can be classified into two types:

– The first type, called type $\mathcal{A}$ in the following, corresponds to the deformations of representations that exist in the classical case $q = 1$. These representations are also called restricted representations since they are also representations of the finite

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