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A Duality for Hopf Algebras and for Subfactors. I

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Dedicated to Masamichi Takesaki on the occasion of his sixtieth birthday

Abstract: We provide a duality between subfactors with finite index, or finite dimensional semisimple Hopf algebras, and a class of C^*-categories of endomorphisms.

1. Introduction

The aim of this work is to provide a duality between subfactors with finite index of an infinite factor M or finite-dimensional (semisimple, complex) Hopf algebras and a class of C^*-categories.

Hereafter we shall restrict ourselves to the case of concrete C^*-categories that are realized by endomorphisms of M [6] and we will provide a general construction of a crossed product algebra. In the sequel of this paper our duality will be formulated in terms of abstract C^*-categories.

Our main technique is index theory for infinite factors [13, 7, 14], sector theory in particular, and we rely on the following ideas. Suppose that a subfactor $N \subset M$ has been constructed, then M becomes equipped with a distinguished sector (an endomorphism up to inner automorphisms) λ, the canonical endomorphism of M into N [17]. The sectors in the irreducible decomposition of $\lambda|_N$ then provide the dual C^*-category.

To give insight to this structure let us recall the simple example of a faithful action α of a finite group G on an infinite factor M with irreducible fixed-point subfactor N. In this case the sectors in the irreducible decomposition of λ furnish the group G

$$\lambda \cong \bigoplus_{g \in G} \alpha_g ,$$

while the restriction of λ to N corresponds to the dual $\hat{G}$ of G

$$\lambda|_N \cong \bigoplus_{\pi \in \hat{G}} d(\pi)\, \varrho_\pi ,$$

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