

# The Second Gelfand–Dickey Bracket as a Bracket on a Poisson–Lie Grassmannian

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*Dedicated to Israel M. Gelfand at his 80<sup>th</sup> birthday*

**Abstract:** We introduce a Poisson structure on a Grassmannian  $\text{Gr}_k(V)$  on which the Poisson–Lie group  $\text{GL}(V)$  acts in a Poisson–Lie way. We discuss the analytic complications connected with the infinite-dimensional case  $V = C^\infty(\mathbb{R})$  and show that an open subset of  $\text{Gr}_k(V)$  with this Poisson structure is isomorphic to the Gelfand–Dickey manifold of differential operators of order  $k$  with the second Gelfand–Dickey bracket. In fact we introduce as a consequence a Poisson–Lie action of an enormous group on the Gelfand–Dickey manifold generalizing (on the semiclassical level) the Sugawara inclusion.

## Contents

<b>0. Introduction</b> . . . . .	94
<b>1. Classical <math>r</math>-matrices and Poisson–Lie structures</b> . . . . .	96
1.1. The classical Yang–Baxter equation . . . . .	96
1.2. Poisson manifolds . . . . .	97
1.3. Skewsymmetric $r$ -matrices . . . . .	98
1.4. The Modified Yang–Baxter equation . . . . .	100
1.5. Homogeneous spaces with Poisson–Lie action . . . . .	102
<b>2. Gelfand–Dickey brackets</b> . . . . .	104
2.0. The second Gelfand–Dickey bracket on the set of differential operators . . . . .	105
2.1. The identification with a Grassmannian . . . . .	107
2.2. A Poisson–Lie algebra of differential operators as a Poisson–Lie subalgebra of $gl$ . . . . .	111
2.3. The periodical case . . . . .	112
2.4. The matrix case . . . . .	114
2.5. A conjecture on quantization: the Kac–Moody case . . . . .	115
2.6. The topological approach . . . . .	116
<b>References</b> . . . . .	119