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Dynamics of Abelian Higgs Vortices in the Near Bogomolny Regime

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Abstract: The aim of this paper is to give an analytical discussion of the dynamics of the Abelian Higgs multi-vortices whose existence was proved by Taubes ([JT82]). For a particular value of a parameter of the theory, λ, called the Higgs self-coupling constant, there is no force between two vortices and there exist static configurations corresponding to vortices centred at any set of points in the plane. This is known as the Bogomolny regime. We will develop some formal asymptotic expansions to describe the dynamics of these multi-vortices for λ close, but not equal to, this critical value. We shall then prove the validity of these asymptotic expansions. These expansions allow us to give a finite dimensional Hamiltonian system which describes the vortex dynamics. The configuration space of this system is the "moduli space" – the space of solutions of the static equations modulo gauge equivalence. The kinetic energy term in the Hamiltonian is obtained from the natural metric on the moduli space given by the L^2 inner product of the tangent vectors. The potential energy gives the intervortex potential which is non-zero when λ is not given by its critical value. Thus the reduced equations for the evolution of the vortex parameters take the form of geodesics, with force terms to express the departure from the Bogomolny regime. The geodesics are geodesics on the moduli space with respect to the metric defined by the L^2 inner product of the tangent vectors, in accordance with Manton's suggestion ([Man82]). This allows an understanding of the two main phenomenological issues – first of all there is the right angle scattering phenomenon, according to which two vortices passing through one another scatter through ninety degrees. Secondly there is the conjecture from numerical calculations that vortices repel for λ greater than the critical value, and attract for λ less than this value. The results of this paper allow a rigorous understanding of the right angle scattering phenomenon ([Sam92, Hit88]) and reduce the question of attraction or repulsion in the near Bogomolny regime to an understanding of the potential energy term in the Hamiltonian ([JR79]).

1. Introduction

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