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Representations of Quantum $so(8)$ and Related Quantum Algebras

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Abstract: We study irreducible representations of the quantum group $U_\varepsilon(so(8))$ when $\varepsilon \in \mathbb{C}^*$ is a primitive l^{th} root of unity. By a theorem of De Concini and Kac, there is a finite number of such representations associated to each point of a complex algebraic variety of dimension 28 and the generic representation has dimension l^{12}.

We give explicit constructions of essentially all the irreducible representations whose dimension is divisible by l^8. In addition, we construct all cyclic representations of minimal dimension. This minimal dimension is l^5, in accordance with a conjecture of De Concini, Kac and Procesi.

1. Introduction

If $\underline{g}$ is finite-dimensional complex simple Lie algebra, there is a well-known family $\{U_q(\underline{g});\ q \in \mathbb{C}^\times\}$ of Hopf algebras over $\mathbb{C}$ which "tend" in a precise sense, to the universal enveloping algebra of $\underline{g}$ as q tends to 1. The algebra $U_q(\underline{g})$ is generated by elements $e_i, f_i, k_i^{\pm 1}$, $i = 1, \ldots, n = rk(\underline{g})$, satisfying certain relations which may be found in Sect. 2.

If q is not a root of unity, the representation theory of $U_q(\underline{g})$ is the "same" as that of $\underline{g}$ [8]. On the other hand, if $q = \varepsilon$ is an l^{th} root of unity, where we assume that l is odd and greater than 1, there are finitely many finite-dimensional irreducible $U_\varepsilon(\underline{g})$-modules associated to every point of a certain complex algebraic veriety of dimension $m = \dim(\underline{g})$ [5]. All such representations have dimension at most $l^{(m-n)/2}$. Although the results of [5] give an adequate parametrization of the set of irreducible representations of $U_\varepsilon(\underline{g})$, they do not give any explicit description of the representations themselves (except in the sl_2 case). It is of interest to give such descriptions, partly to test certain conjectures made in [5 and 6], and also because of certain analogies between the representation theory of $U_\varepsilon(\underline{g})$ and that of $\underline{g}$ over

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