Commun. Math. Phys. 158, 569–644 (1993)

Communications in Mathematical Physics

On Classification of $N = 2$ Supersymmetric Theories

Sergio Cecotti[1], **Cumrun Vafa**[2]

[1] International School for Advanced Studies, SISSA-ISAS, Trieste and I.N.F.N., sez. di Trieste, Trieste, Italy
[2] Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138, USA

Received: 23 November 1992

Abstract: We find a relation between the spectrum of solitons of massive $N = 2$ quantum field theories in $d = 2$ and the scaling dimensions of chiral fields at the conformal point. The condition that the scaling dimensions be real imposes restrictions on the soliton numbers and leads to a classification program for symmetric $N = 2$ conformal theories and their massive deformations in terms of a suitable generalization of Dynkin diagrams (which coincides with the A–D–E Dynkin diagrams for minimal models). The Landau-Ginzburg theories are a proper subset of this classification. In the particular case of LG theories we relate the soliton numbers with intersection of vanishing cycles of the corresponding singularity; the relation between soliton numbers and the scaling dimensions in this particular case is a well known application of Picard-Lefschetz theory.

1. Introduction

Quantum field theories in two dimensions have been under intensive investigation recently in part due to their importance in string theory and in part serving as exactly soluble toy models for quantum field theories in higher dimensions. The interest in studying them for string theory has mostly focused on conformal field theories, i.e., the ones with traceless energy momentum tensor (with only massless excitations). On the other hand, as examples of interesting exactly soluble quantum field theories with interesting S-matrices, the massive ones have been under investigation [1]. In view of the fact that massive QFT's can be viewed as deformation of the conformal theories, it is natural to ask if there is any way to understand properties of conformal theories, by studying the massive analogs. This program has been followed with a spectacular degree of success originating with the work of Zamolodchikov's [2, 3]. The method to relate properties of integrable massive theories to the conformal ones uses thermodynamical Bethe ansatz (TBA). In this way, just by studying the S-matrices of the massive integrable theories one can deduce for example the central charge of the conformal theory.