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Conformal Quantum Field Theory and Half-Sided Modular Inclusions of von-Neumann-Algebras

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Abstract: Let $\mathcal{N}$, $\mathcal{M}$ be von-Neumann-Algebras on a Hilbert space $\mathcal{H}$, Ω a common cyclic and separating vector. Assume Ω to be cyclic and separating also for $\mathcal{N} \cap \mathcal{M}$. Denote by $J_{\mathcal{M}}$, $J_{\mathcal{N}}$ the modular conjugations to $(\mathcal{M}, \Omega)$, $\Delta_{\mathcal{M}}$ and $\Delta_{\mathcal{N}}$ the associated modular operators. If

$$\Delta_{\mathcal{M}}^{-it}(\mathcal{N} \cap \mathcal{M})\Delta_{\mathcal{M}}^{it} \subset (\mathcal{N} \cap \mathcal{M}) \quad \text{for all } t \geqq 0 \,,$$

$$\Delta_{\mathcal{N}}^{it}(\mathcal{N} \cap \mathcal{M})\Delta_{\mathcal{N}}^{-it} \subset (\mathcal{N} \cap \mathcal{M}) \quad \text{for all } t \geqq 0 \,,$$

and

$$J_{\mathcal{M}} \mathcal{N} J_{\mathcal{M}} = \mathcal{N} \,,$$

these data define in a canonical way a conformal quantum field theory on a circle. Conversely, the chiral part of a conformal quantum field theory in two dimensions always yields such data in a natural way.

1. Introduction

It is well known that conformal quantum field theory in two dimensions factor into two chiral conformal theories on the lightrays, see [5]. In the framework of Algebraic Quantum Field Theory, see [6], they are described by a net $\mathcal{A}(I)$ of von-Neumann algebras, indexed by the set $\mathcal{J}$ of proper intervals $I \subset S^1$, with

1. $\mathcal{A}(I) \subset \mathcal{A}(J)$ if $I \subset J$ (isotony)
2. $\mathcal{A}(I) \subset \mathcal{A}(J)'$ if $I \cap J = \emptyset$ (locality),

acting on a Hilbert space $\mathcal{H}$. On $\mathcal{H}$ there is given a strongly continuous unitary positive energy representation U of $Sl(2, \mathbf{R})/\mathbf{Z}_2$ with a unique invariant vacuum vector Ω. The net transforms covariantly under this representation.

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