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Perturbation Theory for the Decay Rate of Eigenfunctions in the Generalized N-body Problem

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Abstract. Simple examples are known where eigenfunctions decay faster than the usual upper bounds would lead one to believe. We develop aspects of the perturbation theory of the decay rate of eigenfunctions as measured by radial exponential weights. We show that generically (in a Baire category sense) eigenfunction decay rates are governed by the lowest threshold.

1. Introduction

There is now an enormous literature on exponential decay of eigenfunctions in the N-body problem. The best known upper bonds are due to Agmon [A1]. If H is the generalized N-body Schrödinger operator (see Sect. II for details), and $E < \Sigma_0(H) = \inf \sigma_{\mathrm{ess}}(H)$, these bounds state that any L^2 solution of $H\psi = E\psi$ satisfies

$$|\psi(x)| \leqq c_\gamma e^{-\gamma\rho_A(x)}, \quad \text{all } \gamma < 1 \ , \tag{1.1}$$

where $\rho_A(x)$ is the distance to the origin in the "Agmon metric" [A1]. If $\psi(x)$ is the unique ground state with eigenvalue $E < \Sigma_0(H)$, it is known [CS] that

$$|\psi(x)| \geqq \tilde{c}_\gamma e^{-\gamma\rho_A(x)}, \quad \text{all } \gamma > 1 \ , \tag{1.2}$$

with $\tilde{c}_\gamma > 0$, but for other eigenfunctions lower bounds are harder to come by. We mention here some results of [FH1, 2] in this direction: Define

$$\alpha_\psi \equiv \sup\{\alpha \geqq 0\colon \exp(\alpha|x|)\psi \in L^2\} \ .$$

Then $\alpha_\psi^2 + E$ is a threshold or $+\infty$. (The possibility $+\infty$ can be eliminated with certain assumptions about the potential which we will not make here.) The set $\mathscr{T}(H)$ of thresholds is a closed countable set to be defined later. Suffice it to say for

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