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Quantization and Representation Theory of Finite W Algebras

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Abstract: In this paper we study the finitely generated algebras underlying W algebras. These so called "finite W algebras" are constructed as Poisson reductions of Kirillov Poisson structures on simple Lie algebras. The inequivalent reductions are labeled by the inequivalent embeddings of sl_2 into the simple Lie algebra in question. For arbitrary embeddings a coordinate free formula for the reduced Poisson structure is derived. We also prove that any finite W algebra can be embedded into the Kirillov Poisson algebra of a (semi)simple Lie algebra (generalized Miura map). Furthermore it is shown that generalized finite Toda systems are reductions of a system describing a free particle moving on a group manifold and that they have finite W symmetry. In the second part we BRST quantize the finite W algebras. The BRST cohomology is calculated using a spectral sequence (which is different from the one used by Feigin and Frenkel). This allows us to quantize all finite W algebras in one stroke. Examples are given. In the last part of the paper we study the representation theory of finite W algebras. It is shown, using a quantum version of the generalized Miura transformation, that the representations of finite W algebras can be constructed from the representations of a certain Lie subalgebra of the original simple Lie algebra. As a byproduct of this we are able to construct the Fock realizations of arbitrary finite W algebras.

1. Introduction

It is only relatively recent that it was realized that nonlinear symmetry algebras play an important role in physics. The discovery of W algebras in Conformal Field theory [1] (see [2] for a recent review) made it clear that they would play an important role in string theory, field theory, integrable systems and the theory of 2D critical phenomena. One reason for their late discovery is that up to now they

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